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# THE THEORY OF ATMOSPHERIC SEEING

by

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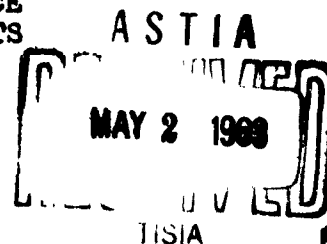
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**CONSIGLIO NAZIONALE DELLE RICERCHE**

**DONALD H. MENZEL**

**The theory of atmospheric seeing**

**R O M A**

**CONSIGLIO NAZIONALE DELLE RICERCHE**

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The objective of this paper is the derivation of formulas to express quantitatively the effects of what astronomers conventionally term «atmospheric seeing». Although the physical principles underlying the phenomenon are well understood, the basic theory has apparently not previously been developed.

Let us start from Snell's well-known law, relating the refractive index,  $\mu$ , and the sine of angle of incidence,  $\theta$ , at a plane boundary.

We have

$$(1) \quad \mu \sin \theta = \text{const.}$$

Logarithmic differentiation gives

$$(2) \quad \frac{d\mu}{\mu} = d \ln \mu = -\cot \theta d\theta.$$

Let  $ds$  denote an element of the incident ray. We have the mathematical relation

$$(3) \quad d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy + \frac{\partial \psi}{\partial z} dz = ds \cdot \nabla \psi = |\nabla \psi| ds \cos \theta,$$

where  $\psi$  is any function. Now let  $\psi = \ln \mu$ . Then (2) shows that the ray impinging on the surface suffers a deflection  $d\theta$ , such that

$$(4) \quad d\theta = -\tan \theta ds \cdot \nabla \ln \mu = -\sin \theta \cdot |\nabla \ln \mu| ds.$$

Equation (4) is more general than equation (1), since the refractive index may vary from point to point and  $\nabla \ln \mu$  may change continually both in magnitude and direction. This equation does not appear in any of the textbooks consulted.

The refractive index of a gas is given by

$$(5) \quad \mu = 1 + (\mu_0 - 1)q;$$

$$(\mu_0 - 1) \times 10^6 =$$

$$= 287.566 + 1.3412/\lambda^2 + 0.03777/\lambda^4,$$

where  $q$  is the ratio between the actual density and the density at standard pressure (760 mm of mercury and temperature 273°K). For air, the quantity  $\mu_0$  depends on the wavelength,  $\lambda$ , here measured in microns. The chromatic effects are generally small, though this part of the problem will be considered later.

From (2) and (4) we conclude that, for light of a given wavelength, the deflection of a light-ray depends only on the density gradient. Near the surface of the earth, the density gradient has a systematic component and a fluctuating component, in both altitude and azimuth. The fluctuating component arises from local heating and cooling effects, changes in the weather, etc. The systematic component depends primarily upon the variation of density and temperature with height. The density will, in general, also depend on the azimuth, primarily as a result of differential heating and associated convection.

As long as we consider path-lengths that are short compared with the radius of the earth (of the order of 1 km versus 6378 km), we may neglect the effects of the earth's curvature and assume that the atmosphere is stratified in plane-parallel layers in the region with which we are concerned. Let  $z$  denote the vertical coordinate. Then by (1) we have

$$(6) \quad d\theta = -\sin\theta \frac{d\ln\mu}{dz} dz.$$

Here  $\theta$  is the zenith distance, the angle between the light-ray and the vertical. Since the refractive index differs only slightly from unity, we may expand  $\ln\mu$  in a MacLaurin series, so that:

$$(7) \quad \ln\mu = (\mu_0 - 1)q,$$

and

$$(8) \quad \frac{d\ln\mu}{dz} = (\mu_0 - 1) \frac{dq}{dz} = -a(\mu_0 - 1)e^{-ah}.$$

Since the total deflection for path-lengths up to a kilometer or so ordinarily turns out

to be small, we may integrate equation (6) keeping the factor  $\sin\theta$  on the right-hand side constant:

$$(9) \quad \Delta\theta = -\sin\theta \frac{d\ln\mu}{dz} l,$$

where  $\Delta\theta$  is the deflection for a total path-length  $l$ . Inserting (8) in this equation we have:

$$(10) \quad \Delta\theta = a(\mu_0 - 1)e^{-ah}(\sin\theta)l.$$

We see that deflection is positive, so that the apparent altitude of a distant object is greater than its true altitude by the amount given by (10). Notice that the density  $\rho_0$  does not enter into equation (10). Inserting numerical values and expressing the deflection in seconds of arc rather than in radians (1 radian = 206,265 seconds of arc), we have

$$(11) \quad \Delta\theta = 6e^{-0.1h}l \sin\theta \text{ seconds of arc.}$$

where  $h$  and  $l$  are measured in kilometers. This result holds for all colors in the visible part of the spectrum, and hence for ordinary white light. For a path-length of 100 meters, at sea level, we have

$$(12) \quad \Delta\theta = 0''.6 \sin\theta.$$

Thus, the maximum deflection occurs for horizontal rays, and amounts to about 0.6 seconds of arc. The seasonal variation of the constant  $a$  is about 10 per cent and according to (10) the deflection will also vary by this amount. As previously noted, the systematic deflection occurs in the vertical plane and is usually zero in azimuth, unless some unusual external condition, such as the presence of a heated dome or wall, tends to set up horizontal temperature gradients.

The foregoing analysis may be extended to include the entire atmosphere including the effects of curvature. The systematic refraction does not necessarily refer to the standard lapse rate, but to the mean or average condition at a given time. The mean trajectory implies a definite variation of temperature and density along  $s$ , even if we do not know it exactly.

We next proceed to evaluate the effect of disturbances or departures from the mean conditions. A mass of relatively hot or cold air between the object and the observer may produce a much larger deflection than the systematic deflection discussed above. This situation would arise if the object itself were hotter or colder than its surroundings. In general, a localized source or sink of heat will give rise to complicated patterns of convection, and it is impossible to obtain from theory alone a detailed description of the resulting deflection and its variation in time. We can, however, get some idea of the magnitude of the effect and give some basic principles for employing statistics to determine the nature of the physical condition of the intervening atmosphere.

Since the air is free to expand or contract, local heating or cooling will alter the pressure only momentarily. The main effect will be to change the density, and we can write for isobaric conditions,

$$(13) \quad \frac{\Delta \rho}{\rho} = - \frac{\Delta T}{T}$$

where  $\Delta \rho$  and  $\Delta T$  indicate respective increments of density and temperature. The atmospheric density and temperature, and hence the refractive index, will vary considerably along the path of a light-ray. If we knew this variation in detail we could, in principle, integrate equation (2) or (4) numerically to obtain the total deflection. Since we cannot predict this variation theoretically, it is more conve-

nient to use in place of equation (1) the equation derived from it by direct application of the theorem of the mean to equation (2) in the form:

$$(14) \quad \cot \theta \Delta \theta = - \frac{\Delta \mu}{\mu}$$

where  $\Delta \theta$  is the total deflection,  $\Delta \mu$  the variation in refractive index from the mean or systematic value previously defined, and  $\theta$  is the average value of the angle between the light-ray and the direction of the density gradient. The mean value of  $\cot \theta$ , over a hemisphere, is unity. Hence,

$$(15) \quad \frac{\Delta \mu}{\mu} = (\mu_0 - 1) \frac{\Delta \rho}{\rho}$$

and

$$(16) \quad \frac{\Delta \mu}{\mu} = - (\mu_0 - 1) \frac{\Delta T}{T}$$

The expected deflection is

$$(17) \quad |\Delta \theta| = (\mu_0 - 1) \frac{\Delta T}{T}$$

For example, if  $\Delta T = 30^\circ \text{C}$ ,  $T = 15^\circ \text{C} = 288^\circ \text{K}$ , then  $\Delta T/T = 0.1$ , approximately, and

$$(18) \quad |\Delta \theta| = 0.00029 \times 0.1 \times 206265 = 6''$$

The values given above for  $\Delta T$  represent conditions that are fairly extreme, except when very cold or very hot sources are involved.

The preceding section gave the deflection resulting from a single temperature discontinuity. In the neighborhood of a relatively hot or cold surface, the situation is more complicated. The surface does not uniformly heat or cool a region in



its neighborhood and a phenomenon known as fibrous convection occurs. Near a hot surface, warm air will rise in fibrous currents, separated by cool regions. Similarly, an analysis of the temperature distribution in the neighborhood of a cold surface would show alternate regions of relatively warm and cool air. Hence a ray of light passing close to a hot or a cold surface will in general intersect a large number of elements of different temperature, and each time it does so it will be deflected. If there are  $N$  such elements in the path, then the total deflection will be of the order of  $N^{1/2}$  times the magnitude of an individual deflection. For example, if an individual deflection is of the order of  $6''$ , as in the preceding section, and there are 100 elements in the path, the total deflection will be of the order of  $60''$ . The image of a point source suffers from blurring or shimmer as a result.

The above result can be derived from arguments analogous to those employed in the problem of the «random walk», which may be stated as follows: If a man walks a distance  $a$ , turns through an arbitrary angle, generally not equal to the first, and continues the process  $N$  times, what is the probability that he will be a distance  $a$  from his point of departure? The result, as WATSON and others have shown, can be expressed in terms of Bessel functions. The optical analogue of this problem, however, is relatively simple, because the angles are not entirely arbitrary and are generally small. The square-root law, familiar in many statistical analyses, applies to give the result previously derived.

A ray of light moving through a uniform atmosphere follows a straight line. If the beam encounters  $N$  discontinuities of the type specified it suffers successive deflections that may be described as random walks when projected upon a plane parallel to the face of the objective lens. Each angular deflection,  $\Delta\theta$ , produces a random motion of magnitude

$$(19) \quad a = l \Delta\theta,$$

where  $l$  is the thickness of the path element. According to the theory of random flights in two dimensions, the probability that the total displacement from the initial undisturbed position after  $N$  deflections, will be at least equal to  $r$  is

$$(20) \quad P = 1 - e^{-r^2/Na^2},$$

on the supposition that the layers are of equal thickness. The probability, therefore, of having a deflection between  $r$  and  $r + dr$  is

$$(21) \quad dP = \frac{2r}{Na^2} e^{-r^2/Na^2} dr.$$

The average total, therefore, is

$$(22) \quad \begin{aligned} \bar{r} &= \int_0^\infty r dP \\ &= \int_0^\infty \pi \bar{N} a/2 = \int_0^\infty \pi \bar{N} l \Delta\theta/2 \\ &= \int_0^\infty \pi \bar{N} l (\mu_0 - 1) \frac{\Delta T}{T} \end{aligned}$$

Analogously,

$$(23) \quad r^2 = Na^2 = \sigma^2 = l^2/N.$$

The presence of a large number of relatively hot and cold elements in the path of a light-ray has a defocussing effect. Extended objects will appear to shimmer or oscillate, and point-sources, or details on extended sources, will be blurred. In the example of the preceding paragraph, a point-source would be spread out into a fuzzy image about  $60''$  in radius, i.e., two minutes in diameter. Astronomers generally term this fuzzy, wavering appearance «poor seeing».

The most serious effects result from fibrous convection like the flow over a hot stove, when the air is not heated uniform-

ly and rises and descends in very small units indeed. Under such conditions,  $N$  can be very large. If the dimension of a unit is  $l$ , and  $L$  is the total path length, we have approximately,

$$(24) \quad N = L/l.$$

Since  $l$  may be as small as 1 cm, if  $L$  is 100 meters,  $N$  may be as great as  $10^4$ . Thus, if  $\Delta T/T$  is only 0.01, representing a departure of about 3°C from the mean, the standard deviation could amount to a full minute of arc.

To study the magnitude of the effect, under extremely severe conditions, I took advantage of an opportunity to observe with a telescope through the exhaust gas from a stationary jet plane. The heated gas, streaming behind the plane, gradually mixed with the atmosphere. I detected the effects of fibrous convection for a large area beyond and above the jet. Local deviations, observed through the telescope, amounted to at least 5 minutes of arc. An observer, seated in a plane well behind the jets, can readily detect the blurring of objects on the ground as seen through the exhaust.

The chance observation of a plane silhouetted against the solar disk also shows the blurring effect of the aerodynamic disturbance for propeller planes as well as for jets. Photographs taken at the Lockheed Solar Observatory demonstrate the phenomenon.

According to equation (5),  $(\mu_0 - 1) \times 10^6$  assumes the following values for various values of  $\lambda$ :

COLOR	$\lambda$ (microns)	$(\mu_0 - 1) \times 10^6$
Ultraviolet .	0.3	292.2
Green . . . .	0.5	270.0
Yellow . . .	0.6	277.0
Red . . . . .	0.7	275.8
Infrared . . .	1.0	274.2

The maximum departure, from infrared to ultraviolet, amounts to 18 units in 280, or about 6 per cent. From green to infrared, the variation is much less, amounting to only 2 per cent.

The values given for  $\sigma$ , in equation (20), which amounted to about 60" or 1 minute of arc for white light for a representative case, would be 64" for ultraviolet and 59" for infrared - scarcely a significant variation.

I divide the problem of seeing into three distinct parts. First, we encounter the internal or instrumental heating, which is particularly hazardous for solar observation because of the large amounts of heat involved. Fibrous convection is particularly dangerous. Three methods of control are available. First, the use of heat sources or sinks (external cooling) to reduce the temperature to the uniform value or, even better, to adjust the temperature along the vertical in a tower telescope so that its logarithmic gradient will be less than the adiabatic value. This type of atmospheric structure is stable and not subject to convection. Second, the use of insulation with fans to break up the fibrous convection. Third, the use of a vacuum or helium to reduce appreciably the value of  $(\mu_0 - 1)$ . All of these are effective. Experience at Sacramento Peak Observatory, however, indicates that, if the second provision concerning insulation is properly fulfilled, we do not have to resort to the use of a vacuum or helium.

The second critical zone refers to seeing above but in the vicinity of the instrument. Here we encounter many well-known problems, such as control of temperature inside the dome, the reduction of heating of a dome by the use of titanium dioxide paint, exhaust of the dome interior, vegetation in the neighborhood, use of towers, and so on.

The ground itself may act as a source of convection. At night, when radiation may cause the temperature of the ground

to fall below the temperature of the air immediately above, the layers tend to be stable. However, during the day it may be higher. Under these circumstances we should expect to find a convection zone close to the ground. One can significantly reduce shimmer and blurring by locating the equipment at some distance above the ground. Even a moderate elevation helps considerably. The most favorable situation occurs when the viewing telescope can be located in a stable region associated with a temperature inversion, for then the optical path contains a minimum of disturbance, as MOR-

TON has done at the Lockheed Solar Observatory.

The third zone is the higher atmosphere from 100 meters or so on up. Atmospheric variations in these levels are seasonal, and also are associated with local weather, and local geography. In view of the fact that such atmospheric regularities are not subject to control, it is fortunate that they are generally less serious than the other two types of disturbance. This occurs because  $l$ , the size of the irregularity, tends to be large.

In conclusion, I wish to express thanks to my colleague, David Layzer, for his critical assistance in the previous analysis.

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#### SUMMARY

This paper expresses in quantitative terms formulae for describing the various phenomena associated with what astronomers conventionally term "atmospheric seeing". This leads to a study of the effect of various convective and inhomogeneous layers on the path of a ray of light. The quantitative results are discussed as applied to three distinct regions of the atmosphere. First is the internal or instrumental heating, particularly hazardous for solar observation. The second critical zone refers to seeing immediately above but in the vicinity of the instrument. The use of temperature inversions as a stabilizing influence may be important, because it will cut down the number of individual convective zones.

The third zone is the higher atmosphere, associated with local weather and local geography. The study involves use of the theory of random flights to describe the total average deflection caused by the passage of the ray through discontinuities of temperature and density.